

White

**Rose
Maths**

Autumn - Block 2

Four Operations

Overview

Small Steps

- ▶ Add whole numbers with more than 4 digits (R)
- ▶ Subtract whole numbers with more than 4 digits (R)
- ▶ Inverse operations (addition and subtraction) (R)
- ▶ Multi-step addition and subtraction problems (R)
- ▶ Add and subtract integers
- ▶ Multiply 4-digits by 1-digit (R)
- ▶ Multiply 2-digits (area model) (R)
- ▶ Multiply 2-digits by 2-digits (R)
- ▶ Multiply 3-digits by 2-digits (R)
- ▶ Multiply up to a 4-digit number by 2-digit number
- ▶ Divide 4-digits by 1-digit (R)
- ▶ Divide with remainders (R)
- ▶ Short division
- ▶ Division using factors

Notes for 2020/21

Year 6 assumes a lot of prior understanding of four operations. A deep understanding of these concepts are essential to help prepare children for secondary education and beyond.

Some children may not have had much practice in the last few months so we've included extended blocks and plenty of recap.

Overview

Small Steps

- ▶ Long division (1)
- ▶ Long division (2)
- ▶ Long division (3)
- ▶ Long division (4)
- ▶ Factors R
- ▶ Common factors
- ▶ Common multiples
- ▶ Primes to 100
- ▶ Squares and cubes
- ▶ Order of operations
- ▶ Mental calculations and estimation
- ▶ Reason from known facts

Notes for 2020/21

Year 6 assumes a lot of prior understanding of four operations. A deep understanding of these concepts are essential to help prepare children for secondary education and beyond.

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Add More than 4-digits

Notes and Guidance

Children will build upon previous learning of column addition. They will now look at numbers with more than four digits and use their place value knowledge to line the numbers up accurately.

Children use a range of manipulatives to demonstrate their understanding and use pictorial representations to support their problem solving.

Mathematical Talk

Will you have to exchange? How do you know which columns will be affected?

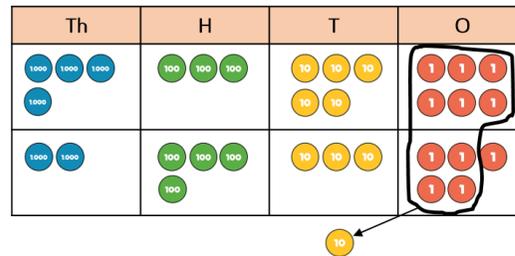
Does it matter that the two numbers don't have the same amount of digits?

Which number goes on top in the calculation? Does it affect the answer?

Varied Fluency



Ron uses place value counters to calculate $4,356 + 2,435$



	Th	H	T	O
	4	3	5	6
+	2	4	3	5
	6	7	9	1

Use Ron's method to calculate:

	3	2	4	6	1
+		4	3	5	2

	4	8	2	7	6
+		5	6	1	3

Jack, Rosie and Eva are playing a computer game. Jack has 3,452 points, Rosie has 4,039 points and Eva has 10,989 points.

How many points do Jack and Rosie have altogether?

How many points do Rosie and Eva have altogether?

How many points do Jack and Eva have altogether?

How many points do Jack, Rosie and Eva have altogether?

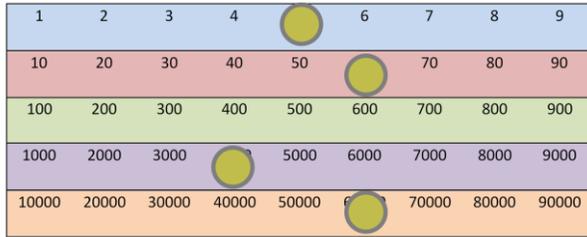
Add More than 4-digits

Reasoning and Problem Solving



Amir is discovering numbers on a Gattegno chart.

He makes this number.



Amir moves one counter three spaces on a horizontal line to create a new number.

When he adds this to his original number he gets 131,130

Which counter did he move?

He moved the counter on the thousands row, he moved it from 4,000 to 7,000

Work out the missing numbers.

	?	4	?	3	?
+	2	?	5	?	2
	7	8	5	2	9

$$54,937 + 23,592 = 78,529$$

Subtract More than 4-digits

Notes and Guidance

Building on Year 4 experience, children use their knowledge of subtracting using the formal column method to subtract numbers with more than four digits. Children will be focusing on exchange and will be concentrating on the correct place value.

It is important that children know when an exchange is and isn't needed. Children need to experience '0' as a place holder.

Mathematical Talk

Why is it important that we start subtracting the smallest place value first?

Does it matter which number goes on top? Why? Will you have to exchange? How do you know which columns will be affected?

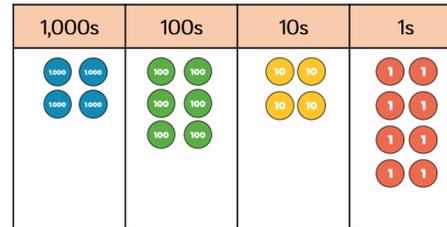
Does it matter that the two numbers don't have the same amount of digits?

Varied Fluency

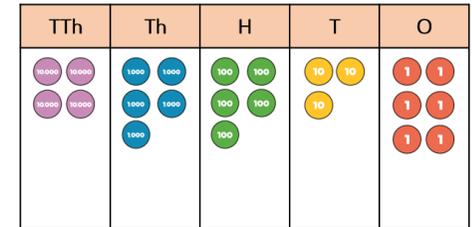


Calculate:

$$4,648 - 2,347$$



$$45,536 - 8,426$$



Represent each problem as a bar model, and solve them.

A plane is flying at 29,456 feet.

During the flight the plane descends 8,896 feet.

What height is the plane now flying at?

Tommy earns £37,506 pounds a year.

Dora earns £22,819 a year.

How much more money does Tommy earn than Dora?

There are 83,065 fans at a football match.

45,927 fans are male. How many fans are female?

Subtract More than 4-digits

Reasoning and Problem Solving



<p>Eva makes a 5-digit number.</p>	<p>Possible answers:</p>
<p>Mo makes a 4-digit number.</p>	<p>9,658 and 14,023</p>
<p>The difference between their numbers is 3,465</p>	<p>12,654 and 8,289</p>
<p>What could their numbers be?</p>	<p>5,635 and 10,000</p>
<p>Etc.</p>	

<p>Rosie completes this subtraction incorrectly.</p>	<p>Rosie did not write down the exchange she made when she exchanged 1 hundred for 10 tens. This means she still had 7 hundreds subtract 6 hundreds when she should have 6 hundreds subtract 6 hundreds. The correct answer is 21,080</p>
$ \begin{array}{r} 28701 \\ - 7621 \\ \hline 21180 \end{array} $	
<p>Explain the mistake to Rosie and correct it for her.</p>	

Inverse Operations

Notes and Guidance

In this small step, children will use their knowledge of addition and subtraction to check their workings to ensure accuracy.

They use the commutative law to see that addition can be done in any order but subtraction cannot.

Mathematical Talk

How can you tell if your answer is sensible?

What is the inverse of addition?

What is the inverse of subtraction?

Varied Fluency


 R

- When calculating $17,468 - 8,947$, which answer gives the corresponding addition question?

$$8,947 + 8,631 = 17,468$$

$$8,947 + 8,521 = 17,468$$

$$8,251 + 8,947 = 17,468$$

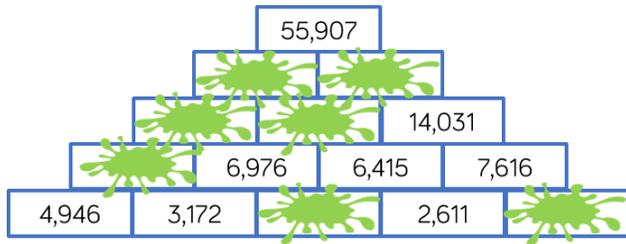
- I'm thinking of a number.
After I add 5,241 and subtract 352, my number is 9,485
What was my original number?
- Eva and Dexter are playing a computer game.
Eva's high score is 8,524
Dexter's high score is greater than Eva's.
The total of both of their scores is 19,384
What is Dexter's high score?

Inverse Operations

Reasoning and Problem Solving



Complete the pyramid using addition and subtraction.



From left to right:

Bottom row:
3,804, 5,005

Second row:
8,118

Third row:
15,094, 13,391

Fourth row:
28,485, 27,422

Mo, Whitney, Teddy and Eva collect marbles.



Mo

I have 1,648 marbles.

I have double the amount of marbles Mo has.



Whitney



Teddy

I have half the amount of marbles Mo has.

In total they have 8,524 marbles between them.

How many does Eva have?

Eva has 2,756 marbles.

Multi-step Problems

Notes and Guidance

In this small step children will be using their knowledge of addition and subtraction to solve multi-step problems.

The problems will appear in different contexts and in different forms i.e. bar models and word problems.

Mathematical Talk

What is the key vocabulary in the question?

What are the key bits of information?

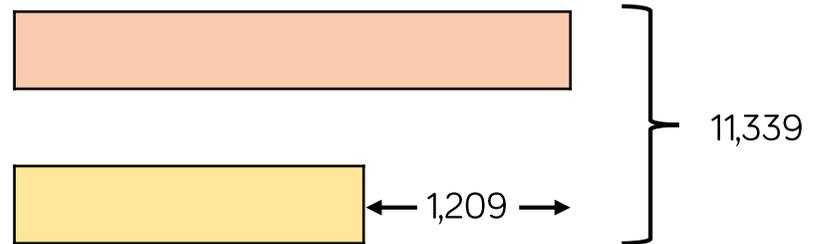
Can we put this information into a model?

Which operations do we need to use?

Varied Fluency

R

-  When Annie opened her book, she saw two numbered pages. The sum of these two pages was 317. What would the next page number be?
-  Adam is twice as old as Barry. Charlie is 3 years younger than Barry. The sum of all their ages is 53. How old is Barry?
-  The sum of two numbers is 11,339. The difference between the same two numbers is 1,209. Use the bar model to help you find the numbers.



Multi-step Problems

Reasoning and Problem Solving



A milkman has 250 bottles of milk.

He collects another 160 from the dairy, and delivers 375 during the day.

How many does he have left?



Tommy

My method:

$$375 - 250 = 125$$

$$125 + 160 = 285$$

Do you agree with Tommy?
Explain why.

Tommy is wrong. He should have added 250 and 160, then subtracted 375 from the answer.

There are 35 bottles of milk remaining.

On Monday, Whitney was paid £114

On Tuesday, she was paid £27 more than on Monday.

On Wednesday, she was paid £27 less than on Monday.

How much was Whitney paid in total?

How many calculations did you do?

Is there a more efficient method?

£342

Children might add 114 and 27, subtract 27 from 114 and then add their numbers.

A more efficient method is to recognise that the '£27 more' and '£27 less' cancel out so they can just multiply £114 by three.

Add & Subtract Integers

Notes and Guidance

Children consolidate their knowledge of column addition and subtraction, reinforcing the language of ‘exchange’ etc. After showing confidence with smaller numbers, children should progress to multi-digit calculations. Children will consider whether the column method is always appropriate e.g. when adding 999, it is easier to add 1,000 then subtract 1

They use these skills to solve multi-step problems in a range of contexts.

Mathematical Talk

What happens when there is more than 9 in a place value column?

Can you make an exchange between columns?

How can we find the missing digits? Can we use the inverse?

Is the column method always the best method?

When should we use mental methods?

Varied Fluency

Calculate.

	3	4	6	2	1
+	2	5	7	3	4
<hr/>					

$$67,832 + 5,258$$

	4	7	6	1	3	2	5
-		9	3	8	0	5	2
<hr/>							

$$834,501 - 299,999$$

A four bedroom house costs £450,000
 A three bedroom house costs £201,000 less.
 How much does the three bedroom house cost?
 What method did you use to find the answer?

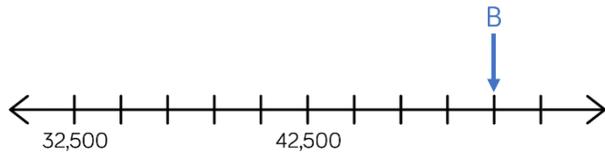
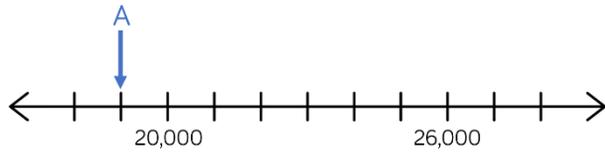
Find the missing digits. What do you notice?

	5	2	2	4	7	?
+	3	?	5	9	0	4
<hr/>						
	9	0	?	3	?	2

Add & Subtract Integers

Reasoning and Problem Solving

Find the difference between A and B.

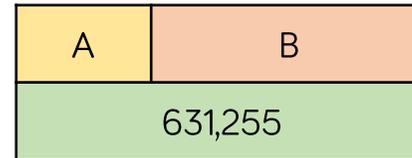


$$A = 19,000$$

$$B = 50,500$$

The difference is 31,500

Here is a bar model.



A is an odd number which rounds to 100,000 to the nearest ten thousand. It has a digit total of 30

B is an even number which rounds to 500,000 to the nearest hundred thousand. It has a digit total of 10

A and B are multiples of 5.

What are possible values of A and B?

Possible answer:

$$A = 99,255$$

$$B = 532,000$$

Multiply 4-digits by 1-digit

Notes and Guidance

Children build on previous steps to represent a 4-digit number multiplied by a 1-digit number using concrete manipulatives. Teachers should be aware of misconceptions arising from using 0 as a place holder in the hundreds, tens or ones column.

Children then move on to explore multiplication with exchange in one, and then more than one column.

Mathematical Talk

Why is it important to set out multiplication using columns?

Explain the value of each digit in your calculation.

How do we show there is nothing in a place value column?

What do we do if there are ten or more counters in a place value column?

Which part of the multiplication is the product?

Varied Fluency



Complete the calculation.

Thousands	Hundreds	Tens	Ones
1000		10 10	1 1 1
1000		10 10	1 1 1
1000		10 10	1 1 1

	T h	H	T	O
	1	0	2	3
×				3

Write the multiplication calculation represented and find the answer.

Thousands	Hundreds	Tens	Ones
1000 1000	100		1 1 1 1 1 1
1000 1000	100		1 1 1 1 1 1

Remember if there are ten or more counters in a column, you need to make an exchange.

Annie earns £1,325 per week. How much would he earn in 4 weeks?

Thousands	Hundreds	Tens	Ones
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1
1000	100 100 100	10 10	1 1 1 1 1

	T h	H	T	O
	1	3	2	5
×				4

Multiply 4-digits by 1-digit

Reasoning and Problem Solving



Alex calculated $1,432 \times 4$

Here is her answer.

	Th	H	T	O
	1	4	3	2
\times				4
	4	16	12	8

$$1,432 \times 4 = 416,128$$

Can you explain what Alex has done wrong?

Alex has not exchanged when she has got 10 or more in the tens and hundreds columns.

Can you work out the missing numbers using the clues?

\times					

$$2,345 \times 5 = 11,725$$

- The 4 digits being multiplied by 5 are consecutive numbers.
- The first 2 digits of the product are the same.
- The fourth and fifth digits of the answer add to make the third.

Multiply 2-digits (Area Model)

Notes and Guidance

Children use Base 10 to represent the area model of multiplication, which will enable them to see the size and scale linked to multiplying.

Children will then move on to representing multiplication more abstractly with place value counters and then numbers.

Mathematical Talk

What are we multiplying?
How can we partition these numbers?

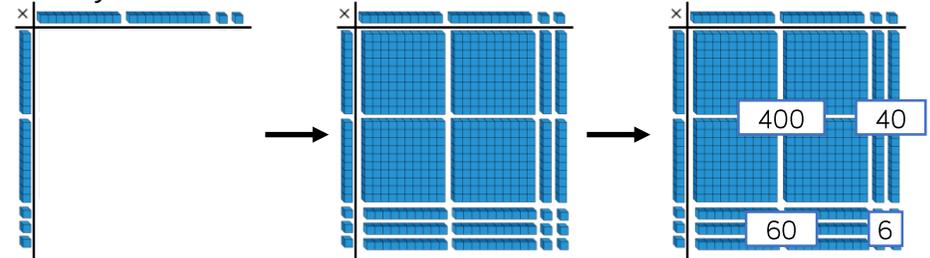
Where can we see 20×20 ?
What does the 40 represent?

What's the same and what's different between the three representations (Base 10, place value counters, grid)?

Varied Fluency

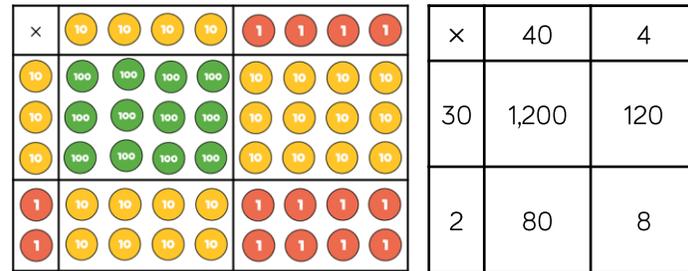


Whitney uses Base 10 to calculate 23×22



How could you adapt your Base 10 model to calculate these:
 32×24 25×32 35×32

Rosie adapts the Base 10 method to calculate 44×32



Compare using place value counters and a grid to calculate:

45×42 52×24 34×43

Multiply 2-digits (Area Model)

Reasoning and Problem Solving



Eva says,



To multiply 23 by 57 I just need to calculate 20×50 and 3×7 and then add the totals.

What mistake has Eva made?
Explain your answer.

Amir hasn't finished his calculation.
Complete the missing information and record the calculation with an answer.

×	40	2
40		
6		

Eva's calculation does not include 20×7 and 50×3
Children can show this with concrete or pictorial representations.

Amir needs 8 more hundreds, $40 \times 40 = 1,600$ and he only has 800
His calculation is $42 \times 46 = 1,932$

Farmer Ron has a field that measures 53 m long and 25 m wide.

Farmer Annie has a field that measures 52 m long and 26 m wide.

Dora thinks that they will have the same area because the numbers have only changed by one digit each.

Do you agree? Prove it.

Dora is wrong. Children may prove this with concrete or pictorial representations.

Multiply 2-digits by 2-digits

Notes and Guidance

Children will move on from the area model and work towards more formal multiplication methods.

They will start by exploring the role of the zero in the column method and understand its importance.

Children should understand what is happening within each step of the calculation process.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and in the second line?

When do we need to make an exchange?

What can we exchange if the product is 42 ones?

If we know what 38×12 is equal to, how else could we work out 39×12 ?

Varied Fluency



Complete the calculation to work out 23×14

		2	3	
x		1	4	
		9	2	(23×4)
		2	3	0 (23×10)

Use this method to calculate:

34×26 58×15 72×35

Complete to solve the calculation.

		4	6	
x		2	7	
		3	2	2 ($_ \times _$)
		9	2	0 ($_ \times _$)

Use this method to calculate:

27×39 46×55 94×49

Calculate:

38×12

39×12

38×11

Multiply 2-digits by 2-digits

Reasoning and Problem Solving



Tommy says,

It is not possible to make 999 by multiplying two 2-digit numbers.



Do you agree?
Explain your answer.

Children may use a trial and error approach during which they'll further develop their multiplication skills. They will find that Tommy is wrong because 27×37 is equal to 999

Amir has multiplied 47 by 36



		4	7
×		3	6
	2	8	2
	1	4	1
	3	2	3

Alex says,



Amir is wrong because the answer should be 1,692 not 323

Who is correct?
What mistake has been made?

Alex is correct. Amir has forgotten to use zero as a place holder when multiplying by 3 tens.

Multiply 3-digits by 2-digits

Notes and Guidance

Children will extend their multiplication skills to multiplying 3-digit numbers by 2-digit numbers. They will use multiplication to find area and solve multi-step problems. Methods previously explored are still useful e.g. using an area model.

Varied Fluency



Complete:

		1	3	2
×			1	4
		5	2	8
	1	3	2	0

(132×4) 264×14 264×28

(132×10)

Use this method to calculate:

What do you notice about your answers?

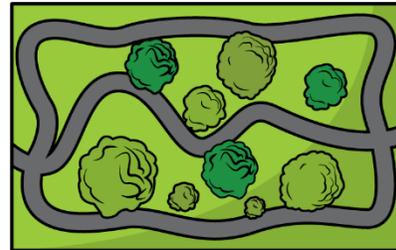
Calculate:

637×24

573×28

573×82

A playground is 128 yards by 73 yards.



Calculate the area of the playground.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and the second line?

When do we need to make an exchange?

What happens if there is an exchange in the last step of the calculation?

Multiply 3-digits by 2-digits

Reasoning and Problem Solving



$$22 \times 111 = 2442$$

$$23 \times 111 = 2553$$

$$24 \times 111 = 2664$$

The pattern stops at up to 28×111 because exchanges need to take place in the addition step.

What do you think the answer to 25×111 will be?

What do you notice?

Does this always work?

Pencils come in boxes of 64
A school bought 270 boxes.
Rulers come in packs of 46
A school bought 720 packs.
How many more rulers were ordered than pencils?



15,840

Here are examples of Dexter's maths work.

			9	8	7
×				7	6
		5	5 ⁹	4 ²	2
		6	6 ⁹	4 ⁰	9
	1	2	8	3	1

			3	2	4	
×				7	8	
				5	9	2
		2	1	3		
	2	1	2	6	8	0
			3	2	7	2

He has made a mistake in each question.

Can you spot it and explain why it's wrong?

Correct each calculation.

In his first calculation, Dexter has forgotten to use a zero when multiplying by 7 tens. It should have been $987 \times 76 = 75,012$

In the second calculation, Dexter has not included his final exchanges. $324 \times 8 = 2,592$
 $324 \times 70 = 22,680$
The final answer should have been 25,272

Multiply 4-digits by 2-digits

Notes and Guidance

Children consolidate their knowledge of column multiplication, multiplying numbers with up to 4 digits by a 2-digit number. It may be useful to revise multiplication by a single digit first, and then 2- and 3- digit numbers before moving on when ready to the largest calculations.

They use these skills to solve multi-step problems in a range of contexts.

Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

Can the inverse operation be used?

Is there a different strategy that you could use?

Varied Fluency

Calculate.

	4	2	6	7
×			3	4

	3	0	4	6
×			7	3

$$5,734 \times 26$$

Jack made cookies for a bake sale. He made 345 cookies. The recipe says that he should have 17 raisins in each cookie.

How many raisins did he use altogether?

Work out the missing number.

$$6 \times 35 = \underline{\quad} \times 5$$

Multiply 4-digits by 2-digits

Reasoning and Problem Solving

True or False?

- $5,463 \times 18 = 18 \times 5,463$
- I can find the answer to $1,100 \times 28$ by calculating $1,100 \times 30$ and subtracting 2 lots of 1,100
- $702 \times 9 = 701 \times 10$

True

True

False



Place the digits in the boxes to make the largest product.

	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
×	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

$$\begin{array}{r}
 8432 \\
 \times 75 \\
 \hline
 632000
 \end{array}$$

Divide 4-digits by 1-digit

Notes and Guidance

Children use their knowledge from Year 4 of dividing 3-digits numbers by a 1-digit number to divide up to 4-digit numbers by a 1-digit number.

They use place value counters to partition their number and then group to develop their understanding of the short division method.

Mathematical Talk

- How many groups of 4 thousands are there in 4 thousands?
- How many groups of 4 hundreds are there in 8 hundreds?
- How many groups of 4 tens are there in 9 tens?
- What can we do with the remaining ten?
- How many groups of 4 ones are there in 12 ones?
- Do I need to solve both calculations to compare the divisions?

Varied Fluency R

Here is a method to calculate 4,892 divided by 4 using place value counters and short division.

Thousands	Hundreds	Tens	Ones
4	8	9	2

	1	2	2	3
4	4	8	9	2

Use this method to calculate:

$6,610 \div 5$

$2,472 \div 3$

$9,360 \div 4$

Mr Porter has saved £8,934
He shares it equally between his three grandchildren.
How much do they each receive?

Use $<$, $>$ or $=$ to make the statements correct.

$3,495 \div 5$ $3,495 \div 3$

$8,064 \div 7$ $9,198 \div 7$

$7,428 \div 4$ $5,685 \div 5$

Divide 4-digits by 1-digit

Reasoning and Problem Solving



Jack is calculating $2,240 \div 7$

He says you can't do it because 7 is larger than all of the digits in the number.

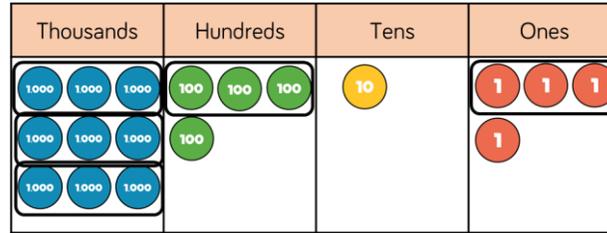
Do you agree with Jack?
Explain your answer.

Jack is incorrect. You can exchange between columns. You can't make a group of 7 thousands out of 2 thousand, but you can make groups of 7 hundreds out of 22 hundreds.

The answer is 320

Spot the Mistake

Explain and correct the working.



	3	1	0	1
3	9	4	1	4

There is no exchanging between columns within the calculation. The final answer should have been 3,138

Divide with Remainders

Notes and Guidance

Children continue to use place value counters to partition and then group their number to further develop their understanding of the short division method.

They start to focus on remainders and build on their learning from Year 4 to understand remainders in context. They do not represent their remainder as a fraction at this point.

Mathematical Talk

If we can't make a group in this column, what do we do?

What happens if we can't group the ones equally?

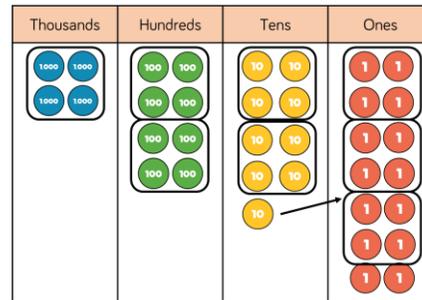
In this number story, what does the remainder mean?

When would we round the remainder up or down?

In which context would we just focus on the remainder?

Varied Fluency R

Here is a method to solve 4,894 divided by 4 using place value counters and short division.



	1	2	2	3		
4	4	8	9	¹ 4		r2

Use this method to calculate:

$6,613 \div 5$

$2,471 \div 3$

$9,363 \div 4$

Muffins are packed in trays of 6 in a factory. In one day, the factory makes 5,623 muffins. How many trays do they need? How many trays will be full? Why are your answers different?

- For the calculation $8,035 \div 4$
- Write a number story where you round the remainder up.
 - Write a number story where you round the remainder down.
 - Write a number story where you have to find the remainder.

Divide with Remainders

Reasoning and Problem Solving



I am thinking of a 3-digit number.

When it is divided by 9, the remainder is 3

When it is divided by 2, the remainder is 1

When it is divided by 5, the remainder is 4

What is my number?

Possible answers:

129	219
309	399
489	579
669	759
849	939

Encourage children to think about the properties of numbers that work for each individual statement. This will help decide the best starting point.

Always, Sometimes,

A three-digit number made of consecutive descending digits divided by the next descending digit always has a remainder of 1

$$765 \div 4 = 191 \text{ remainder } 1$$

How many possible examples can you find?

Sometimes

Possible answers:

$432 \div 1 = 432 \text{ r } 0$
 $543 \div 2 = 271 \text{ r } 1$
 $654 \div 3 = 218 \text{ r } 0$
 $765 \div 4 = 191 \text{ r } 1$
 $876 \div 5 = 175 \text{ r } 1$
 $987 \div 6 = 164 \text{ r } 3$

Short Division

Notes and Guidance

Children build on their understanding of dividing up to 4-digits by 1-digit by now dividing by up to 2-digits. They use the short division method and focus on the grouping structure of division. Teachers may encourage children to list multiples of the divisor (number that we are dividing by) to help them solve the division more easily. Children should experience contexts where the answer “4 r 1” means both 4 complete boxes or 5 boxes will be needed.

Mathematical Talk

In the hundreds column, how many groups of 5 are in 7? Are there are any hundreds remaining? What do we do next?

In the thousands column, there are no groups of three in 1. What do we do?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

Calculate using short division.

5	7	2	5
---	---	---	---

3	1	9	3	8
---	---	---	---	---

1	2	6	0	3	6
---	---	---	---	---	---

$$3,612 \div 14$$

List the multiples of the divisors to help you calculate.

A limousine company allows 14 people per limousine.

How many limousines are needed for 230 people?

Year 6 has 2,356 pencil crayons for the year.

They put them in bundles, with 12 in each bundle.

How many complete bundles can be made?

Short Division

Reasoning and Problem Solving

Find the missing digits.

$$\begin{array}{r} 041\boxed{4}r3 \\ 4\overline{)1\boxed{6}59} \end{array}$$

$$\begin{array}{r} 041\boxed{4}r3 \\ 4\overline{)1\boxed{6}59} \end{array}$$

Here are two calculations.

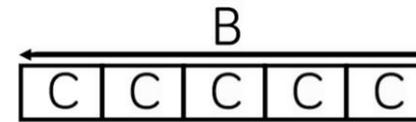
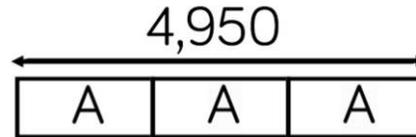
$$A = 396 \div 11$$

$$B = 832 \div 13$$

$$\begin{aligned} 396 \div 11 &= 36 \\ 832 \div 13 &= 64 \\ 64 - 36 &= 28 \end{aligned}$$

Find the difference between A and B.

Work out the value of C.
(The bar models are not drawn to scale)



$$\begin{aligned} 4,950 \div 3 &= \\ 1,650 \end{aligned}$$

$$1,650 \div 3 = 550$$

$$550 \div 5 = 110$$

Division using Factors

Notes and Guidance

Children use their number sense, specifically their knowledge of factors, to be able to see relationships between the dividend (number being divided) and the divisor (number that the dividend is being divided by).

Beginning with multiples of 10 will allow children to see these relationships, before moving to other multiples.

Mathematical Talk

What is a factor?

How does using factor pairs help us to answer division questions?

Do you notice any patterns?

Does using factor pairs always work?

Is there more than one way to solve a calculation using factor pairs?

What methods can be used to check your working out?

Varied Fluency

Calculate $780 \div 20$

Now calculate $780 \div 10 \div 2$

What do you notice? Why does this work?

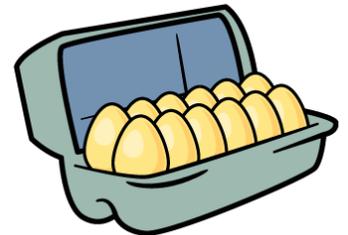
Use the same method to calculate $480 \div 60$

Use factors to help you calculate.

$$4,320 \div 15$$

Eggs are put into boxes.
Each box holds 12 eggs.
A farmer has 648 eggs that need to go in the boxes.

How many boxes will he fill?



Division using Factors

Reasoning and Problem Solving

<p>Calculate:</p> <ul style="list-style-type: none"> • $1,248 \div 48$ • $1,248 \div 24$ • $1,248 \div 12$ <p>What did you do each time? What was your strategy? What do you notice? Why?</p>	<p>26 52 104</p> <p>Children should recognise that when the dividend is halved, the answer (quotient) is doubled.</p>
<p>Tommy says,</p>  <div style="border: 1px solid blue; border-radius: 15px; padding: 10px; display: inline-block;"> <p>To calculate $4,320 \div 15$ I will first divide 4,320 by 5 then divide the answer by 10</p> </div> <p>Do you agree? Explain why.</p>	<p>Tommy is wrong: he has partitioned 15 when he should have used factor pairs. He could have used factor pairs 5 and 3 and divided by 5 then 3 (or 3 then 5).</p>

<p>Class 6 are calculating $7,848 \div 24$</p> <p>The children decide which factor pairs to use. Here are some of their suggestions:</p> <ul style="list-style-type: none"> • 2 and 12 • 1 and 24 • 4 and 6 • 10 and 14 <p>Which will not give them the correct answer? Why?</p> <p>Use the correct factor pairs to calculate the answer. Is the answer the same each time?</p> <p>Which factor pair would be the least efficient to use? Why?</p>	<p>10 and 14 is incorrect because they are not factors of 24 (to get 10 and 14, 24 has been partitioned).</p> <p>The correct answer is 327</p> <p>Children should get the same answer using all 3 factor pairs methods.</p> <p>Using the factor pair of 1 and 24 is the least efficient.</p>
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Long Division (1)

Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number.

They divide 3-digit numbers by a 2-digit number without remainders, starting with a more expanded method (with multiples shown), before progressing to the more formal long division method.

Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

Varied Fluency



		0	3	6
1	2	4	3	2
	–	3	6	0
			7	2
	–		7	2
				0

(x30)

(x6)

Multiples of 12:

- 12 × 1 = 12
- 12 × 2 = 24
- 12 × 3 = 36
- 12 × 4 = 48
- 12 × 5 = 60
- 12 × 6 = 72
- 12 × 7 = 84
- 12 × 8 = 96
- 12 × 7 = 108
- 12 × 10 = 120

Use this method to calculate:

765 ÷ 17

450 ÷ 15

702 ÷ 18



		0	3	6
1	2	4	3	2
	–	3	6	↓
			7	2
	–		7	2
				0

Use the long division method to calculate:

- 836 ÷ 11
- 798 ÷ 14
- 608 ÷ 19

Long Division (1)

Reasoning and Problem Solving

Odd One Out

Which is the odd one out?
Explain your answer.

$$512 \div 16$$

$$672 \div 21$$

$$792 \div 24$$

$792 \div 24 = 33$ so this is the odd one out as the other two give an answer of 32

Spot the Mistake

$$855 \div 15 =$$

		0	5	1	0	
1	5	8	5	5		
	–	7	5		(× 4)	
		1	0	5		
	–	1	0	5	(× 10)	
				0		

The mistake is that $105 \div 15$ is not equal to 10

$105 \div 15 = 7$ so the answer to the calculation is 57

Long Division (2)

Notes and Guidance

Building on using long division with 3-digit numbers, children divide 4-digit numbers by 2-digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

Varied Fluency

Here is a division method.

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400)
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

Use this method to calculate:

$$2,208 \div 16$$

$$1,755 \div 45$$

$$1,536 \div 16$$

There are 1,989 footballers in a tournament. Each team has 11 players and 2 substitutes. How many teams are there in the tournament?

Long Division (2)

Reasoning and Problem Solving

Which calculation is harder?

$$1,950 \div 13$$

$$1,950 \div 15$$

Explain why.

Dividing by 13 is harder because 13 is prime so we cannot use factor knowledge to factorise it into smaller parts. The 13 times table is harder than the 15 times table because the 15 times table is related to the 5 times table whereas the 13 times table is not related to a more common times table (because 13 is prime).

$$6,120 \div 17 = 360$$

Explain how to use this fact to find



$$6,480 \div \text{★} = 360$$

6,480 is 360 more than 6,120, so there is 1 group of 360 more.

Therefore, there are 18 groups of 360, so the answer is 18

Long Division (3)

Notes and Guidance

Children now divide using long division where answers have remainders. After dividing, they check that the remainder is smaller than the divisor.

Children start to understand how to interpret the remainder e.g. $380 \div 12 = 31 \text{ r } 8$ could mean **31 full packs, or 32 packs needed depending on context.**

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

Tommy uses this method to calculate 372 divided by 15. He has used his knowledge of multiples to help.

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

- $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- $4 \times 15 = 60$
- $5 \times 15 = 75$
- $10 \times 15 = 150$

Use this method to calculate:

$271 \div 17$

$623 \div 21$

$842 \div 32$

A school needs to buy 380 biscuits for parents' evening. Biscuits are sold in packs of 12

How many packets will the school need to buy?

Long Division (3)

Reasoning and Problem Solving

Here are two calculation cards.

$$A = 396 \div 11$$

$$B = 832 \div 11$$

Whitney thinks there won't be a remainder for either calculation because 396 and 832 are both multiples of 11

Rosie disagrees, she has done the written calculations and says **one** of them has a remainder.

Who is correct? Explain your answer.

Rosie is correct because 832 is not a multiple of 11

$$396 \div 11 = 36$$

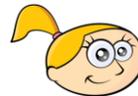
$$832 \div 11 = 75 \text{ r } 7$$

576 children and 32 adults need transport for a school trip. A coach holds 55 people.



Dora

We need 10 coaches.



Eva

We need 11 coaches.



Alex

We need 12 coaches.

Who is correct? Explain how you know.

How many spare seats will there be?

Alex is correct.

There are 608 people altogether, $608 \div 55 = 11 \text{ r } 3$, so 12 coaches are needed.

On 12 coaches there will be 660 seats, because $55 \times 12 = 660$
 $660 - 608 = 52$ spare seats.

Long Division (4)

Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that it is not applicable.

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

Does the remainder need to be rounded up or down?

Varied Fluency

Amir used this method to calculate 1,426 divided by 13

			1	0	9	r	9		
1	3	1	4	2	6				
	–	1	3	0	0			(× 100)	
			1	2	6				
	–		1	1	7			(× 9)	
					9				

Use this method to calculate:

$2,637 \div 16$

$4,453 \div 22$

$4,203 \div 18$

A large bakery produces 7,849 biscuits in a day which are packed in boxes. Each box holds 64 biscuits.

How many boxes are needed so all the biscuits are in a box?

Long Division (4)

Reasoning and Problem Solving

Class 6 are calculating three thousand, six hundred and thirty-three divided by twelve.

Rosie says that she knows there will be a remainder without calculating.

Is she correct?
Explain your answer.

What is the remainder?

Rosie is correct because 3,633 is odd and 12 is even, and all multiples of 12 are even because 12 is even.

$$3,633 \div 12 = 302 \text{ r } 9, \text{ so the remainder is } 9$$

Which numbers up to 20 can 4,236 be divided by without having a remainder?

What do you notice about all the numbers?

1, 2, 3, 4, 6, 12

They are all factors of 12

Factors

Notes and Guidance

Children understand the relationship between multiplication and division and use arrays to show the relationship between them. Children learn that factors of a number multiply together to give that number, meaning that factors come in pairs. Factors are the whole numbers that you multiply together to get another whole number (factor \times factor = product).

Mathematical Talk

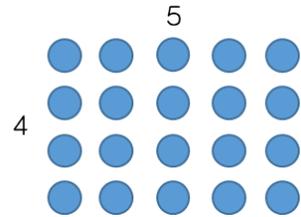
How can you work in a systematic way to prove you have found all the factors?

Do factors always come in pairs?

How can we use our multiplication and division facts to find factors?

Varied Fluency R

If you have twenty counters, how many different ways of arranging them can you find?



How many factors of twenty have you found by arranging your counters in different arrays?

Circle the factors of 60

- 9, 6, 8, 4, 12, 5, 60, 15, 45

Which factors of 60 are not shown?

Fill in the missing factors of 24

$1 \times \underline{\quad}$ $\underline{\quad} \times 12$

$3 \times \underline{\quad}$ $\underline{\quad} \times \underline{\quad}$

What do you notice about the order of the factors?

Use this method to find the factors of 42

Factors

Reasoning and Problem Solving



Here is Annie's method for finding factor pairs of 36

1	36
2	18
3	12
4	9
5	X
6	6

When do you put a cross next to a number?

How many factors does 36 have?

Use Annie's method to find all the factors of 64

If it is not a factor, put a cross.

36 has 9 factors.

Factors of 64:

1	64
2	32
3	X
4	16
5	X
6	X
7	X
8	8

Always, Sometimes, Never

- An even number has an even amount of factors.
- An odd number has an odd amount of factors.

Sometimes, e.g. 6 has four factors but 36 has nine.

Sometimes, e.g. 21 has four factors but 25 has three.

True or False?

The bigger the number, the more factors it has.

False. For example, 12 has 6 factors but 13 only has 2

Common Factors

Notes and Guidance

Children find the common factors of two numbers.

Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged.

They can show their results using Venn diagrams and tables.

Mathematical Talk

How do you know you have found all the factors of a given number?

Have you used a systematic approach?

Can you explain your system to a partner?

How does a Venn diagram show common factors?

Where are the common factors?

Varied Fluency

Find the common factors of each pair of numbers.

24 and 36

20 and 30

28 and 45

Which number's factors make it the odd one out?

12, 30, 54, 42, 32, 48

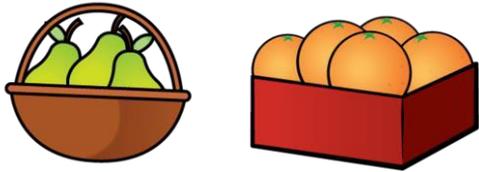
Can you explain why?

Two numbers have common factors of 4 and 9
What could the numbers be?

Common Factors

Reasoning and Problem Solving

There are 49 pears and 56 oranges.



They need to be put into baskets of pears and baskets of oranges with an equal number of fruit in each basket.

Amir says,



There will be 8 pieces of fruit in each basket.

Jack says,

There will be 7 pieces of fruit in each basket.



Who is correct? Explain how you know.

Jack is correct. There will be seven pieces of fruit in each basket because 7 is a common factor of 49 and 56

Tommy has two pieces of string.

One is 160 cm long and the other is 200 cm long.

He cuts them into pieces of equal length.

What are the possible lengths the pieces of string could be?

The possible lengths are: 2, 4, 5, 8, 10, 20 and 40 cm.

Dora has 32 football cards that she is giving away to his friends.

She shares them equally between her friends.

How many friends could Dora have?

Dora could have 1, 2, 4, 8, 16 or 32 friends.

Common Multiples

Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use visual representations to support their thinking.

They also use abstract methods to calculate multiples, including using numbers outside of those known in times table facts.

Mathematical Talk

Is the lowest common multiple of a pair of numbers always the product of them?

Can you think of any strategies to work out the lowest common multiples of different numbers?

When do numbers have common multiples that are lower than their product?

Varied Fluency

- On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5

What common multiple of 7 and 5 do you find?

Use this number to find other common multiples of 7 and 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- List 5 common multiples of 4 and 3
- Alex and Eva play football at the same local football pitches. Alex plays every 4 days and Eva plays every 6 days.

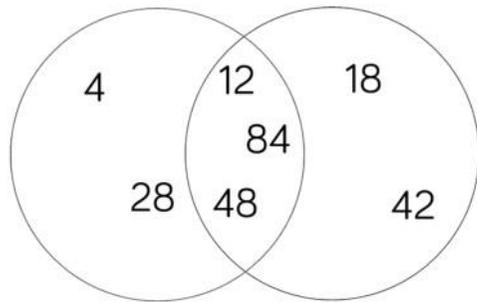
They both played football today.

After a fortnight, how many times will they have played football on the same day?

Common Multiples

Reasoning and Problem Solving

Work out the headings for the Venn diagram.



Add in one more number to each section.

Can you find a square number that will go in the middle section of the Venn diagram?

Multiples of 4
Multiples of 6

144 is a square number that can go in the middle.

Annie is double her sister's age.

They are both older than 20 but younger than 50

Their ages are both multiples of 7

What are their ages?

Annie is 42 and her sister is 21

A train starts running from Leeds to York at 7am.

The last train leaves at midnight.

Platform 1 has a train leaving from it every 12 minutes.

Platform 2 has one leaving from it every 5 minutes.

How many times in the day would there be a train leaving from both platforms at the same time?

18 times

Primes to 100

Notes and Guidance

Building on their learning in year 5, children should know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

They should be able to use their understanding of prime numbers to work out whether or not numbers up to 100 are prime. Using primes, they break a number down into its prime factors.

Mathematical Talk

What is a prime number?

What is a composite number?

How many factors does a prime number have?

Are all prime numbers odd?

Why is 1 not a prime number?

Why is 2 a prime number?

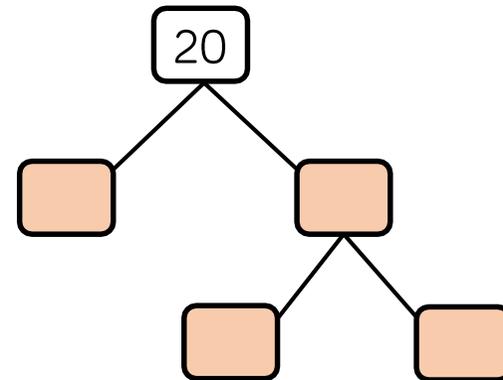
Varied Fluency

List all of the prime numbers between 10 and 30

The sum of two prime numbers is 36

What are the numbers?

All numbers can be broken down into prime factors. A prime factor tree can help us find them. Complete the prime factor tree for 20



Primes to 100

Reasoning and Problem Solving

Use the clues to work out the number. 15

- It is greater than 10
- It is an odd number
- It is not a prime number
- It is less than 25
- It is a factor of 60

Shade in the multiples of 6 on a 100 square.

What do you notice about the numbers either side of every multiple of 6?

Eva says,



I noticed there is always a prime number next to a multiple of 6

Is she correct?

Both numbers are always odd.

Yes, Eva is correct because at least one of the numbers either side of a multiple of 6 is always prime for numbers up to 100

Square & Cube Numbers

Notes and Guidance

Children have identified square and cube numbers previously and now explore the relationship between them, and solve problems involving them. They need to experience sorting the numbers into different diagrams and look for patterns and relationships. They explore general statements regarding square and cube numbers. This step is a good opportunity to practise efficient mental methods of calculation.

Mathematical Talk

What do you notice about the sequence of square numbers?
 What do you notice about the sequence of cube numbers?
 Explore the pattern of the difference between the numbers.

Varied Fluency

- Use $<$, $>$ or $=$ to make the statements correct.
 - 3 cubed 4 squared
 - 8 squared 4 cubed
 - 11 squared 5 cubed

This table shows square and cube numbers. Complete the table. Explain the relationships you can see between the numbers.

		1			1
					8
	3×3		3^3		27
	4×4			$4 \times 4 \times 4$	
		25	5^3		
				$6 \times 6 \times 6$	
8^2					

- $___ + 35 = 99$
- $210 - ___ = 41$
- Which square numbers are missing from the calculations?

Square & Cube Numbers

Reasoning and Problem Solving

Place 5 odd and 5 even numbers in the table.

	Not Cubed	Cubed
Over 100		
100 or less		

Possible cube numbers to use:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000

Shade in all the square numbers on a 100 square.

Now shade in multiples of 4

What do you notice?

Square numbers are always either a multiple of 4 or 1 more than a multiple of 4

Jack says,



The smallest number that is both a square number and a cube number is 64

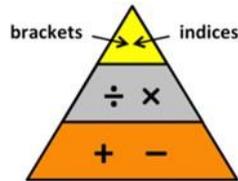
Do you agree with Jack? Explain why you agree or disagree.

Jack is incorrect. 1 is the smallest number that is both a square number ($1^2 = 1$) and cube number ($1^3 = 1$).

Order of Operations

Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. Children will learn that, in mixed operation calculations, calculations are not carried out from left to right. Children learn the convention that when there is no operation sign written this means multiply e.g. $4(2 + 1)$ means $4 \times (2 + 1)$. This image is useful when teaching the order of operations.



Mathematical Talk

Does it make a difference if you change the order in a mixed operation calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

Why?

Varied Fluency

- Alex has 7 bags with 5 sweets in each bag. She adds one more sweet to each bag. Which calculation will work out how many sweets she now has in total? Explain your answer.

$$7 \times (5 + 1)$$

$$7 \times 5 + 1$$

- Teddy has completed this calculation and got an answer of 5

$$14 - 4 \times 2 \div 4 = 5$$

Explain and correct his error.

- Add brackets and missing numbers to make the calculations correct.

$$6 + \underline{\quad} \times 5 = 30$$

$$25 - 6 \times \underline{\quad} = 38$$

Order of Operations

Reasoning and Problem Solving

Countdown

Big numbers: 25, 50, 75, 100

Small numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Children randomly select 6 numbers.

Reveal a target number.

Children aim to make the target number ensuring they can write it as a single calculation using order of operations.

Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use:

- One operation
- Two operations with no brackets
- Two operations with brackets

Possible solutions:

$$58 - 34 = 24$$

$$58 + 3 \times 4 = 60$$

$$5(8 - 3) + 4 = 29$$

Mental Calculations

Notes and Guidance

We have included this small step separately to ensure that teachers emphasise this important skill. Discussions with children around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when more efficient mental strategies should be used.

Mathematical Talk

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

Varied Fluency

- How could you change the order of these calculations to be able to perform them mentally?

$$50 \times 16 \times 2$$

$$30 \times 12 \times 2$$

$$4 \times 17 \times 25$$

- Mo wants to buy a t-shirt for £9.99, socks for £1.49 and a belt for £8.99
He has £22 in his wallet.
How could he quickly check if he has enough money?



What number do you estimate is shown by arrow B when:

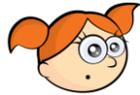
- $A = 0$ and $C = 1,000$
- $A = 30$ and $C = 150$
- $A = -7$ and $C = 17$
- $A = 1$ and $C = 2$
- $A = 1,000$ and $C = 100,000$

Mental Calculations

Reasoning and Problem Solving

Class 6 are calculating the total of 3,912 and 3,888

Alex says,



We can just double 3,900

Is Alex correct? Explain.

Alex is correct because 3,912 is 12 more than 3,900 and 3,888 is 12 less than 3,900

$$3,900 \times 2 = 7,800$$

$$2,000 - 1,287$$

Here are three different strategies for this subtraction calculation:



Dora

I used the column method.



Tommy

I used my number bonds from 87 to 100 then from 1,300 to 2,000



Jack

I subtracted one from each number and then used the column method.

Whose method is most efficient?

Children share their ideas. Discuss how Dora's method is inefficient for this calculation because of the need to make multiple exchanges.

Jack's method is known as the 'constant difference' method and avoids exchanging.

Reason from Known Facts

Notes and Guidance

Children should use known facts from one calculation to determine the answer of another similar calculation without starting afresh.

They should use reasoning and apply their understanding of commutativity and inverse operations.

Mathematical Talk

What is the inverse?

When do you use the inverse?

How can we use multiplication/division facts to help us answer similar questions?

Varied Fluency

Complete.

$$70 \div \underline{\quad} = 7 \quad 3.5 \times 10 = \underline{\quad}$$

$$70 \div \underline{\quad} = 3.5 \quad \underline{\quad} = 3.5 \times 20$$

$$70 \div \underline{\quad} = 14 \quad \underline{\quad} = 3.5 \times 2$$

Make a similar set of calculations using $90 \div 2 = 45$

Complete $5,138 \div 14 = 367$

Use this to calculate 15×367

Complete $14 \times 8 = 112$

Use this to calculate:

- 1.4×8
- 9×14

Reason from Known Facts

Reasoning and Problem Solving

$$3,565 + 2,250 = 5,815$$

Use this calculation to decide if the following calculations are true or false.

True or False?

$$4,565 + 1,250 = 5,815$$

True

$$5,815 - 2,250 = 3,565$$

True

$$4,815 - 2,565 = 2,250$$

True

$$3,595 + 2,220 = 5,845$$

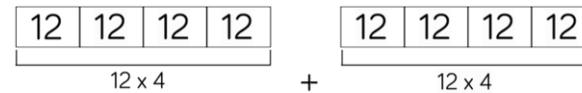
False

Which calculations will give an answer that is the same as the product of 12 and 8?

$$3 \times 4 \times 8$$

$$12 \times 4 \times 2$$

$$2 \times 10 \times 8$$



The product of 12 and 8 is 96

The 1st and 2nd calculations give an answer of 96
 In the 1st calculation 12 has been factorised into 3 and 4, and in the 2nd calculation 8 has been factorised into 4 and 2

The third calculation gives an answer of 160